Finance 799 Project Report

On

Study of the Extremal Dependence Structure in Financial Markets
Using Extreme Value Theory

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Submitted to

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November 6, 2008

1. Introduction

Extremal events, by definition, are those events that rarely happen. Statistically speaking, they only appear in the tails of probability distributions. Extremal events in the financial world can take a variety of different forms, such as the stock market crash as we are experiencing, major defaults in credit risk analysis, or the collapse of risky asset prices. In the conventional statistical analysis, we treat these extremal events as outliers, and leave them out of our views most of the time. However, what happened recently in the financial market highlighted the importance of extremal events in asset pricing, portfolio choice, and risk management.

Extreme Value Theory (EVT), in many cases, is the natural and most efficient recipe for modeling these extremal events. EVT refers to a well-established body of theory that is capable of predicting the occurrence of extremal events, outside the range of available data (Embrechts, Kluppelberg et al. 1999). During the past decade, EVT has experienced fast development in the research fields of Actuarial Science, Risk Management, and Quality Engineering. As evidence, EVT is documented in a great volume of literature articles and books. For instance, (Diebold, Schuermann et al. 1998; Neftci 2000; Zhang 2005) discuss EVT for the univariate cases, while (Tawn 1990; Smith and Weissman 1996; Starica 1999; Starica 2000; Bouye 2002; Buhl, Reich et al. 2002; Mashal and Zeevi 2002; Smith 2003; Heffernan and Tawn 2004; Zhang and Smith 2004) extend the discussion of EVT to the multivariate cases. The encyclopedic (Embrechts, Kluppelberg et al. 1999) offers a comprehensive review of both EVT theory and its applications with an emphasis on the financial markets.

Among these researches, the multivariate version is particularly useful in the context of financial studies, since almost all financial applications involve more than one component, which can only be addressed through a multivariate framework. When all the constituents of a financial application, for instance, different pricing factors for a portfolio, reach their extremal levels, studying their relationships, namely, the extremal, asymptotic or tail dependence structure, provides valuable information about the tail behavior of these constituents. Recently, Zhang (Zhang 2002; Zhang 2003; Zhang 2005; Zhang and Huang 2006; Zhang 2008) and Chamu (Chamu 2005) developed rigorous estimation theory for the Multivariate Maxima of Moving Maxima (M4) model proposed by (Smith and Weissman 1996), and applied such theory to model practical applications in the areas of Finance, Environmental Science and Telecommunications. More importantly, these articles lay out a coherent testing strategy, limiting distribution, as well as modeling framework. The model in this study is implemented based on these new developments, with the focus on the tail dependence testing and computation of a tail dependence measure. More specifically, the multivariate extreme value framework proposed by (Zhang 2003; Zhang 2005; Zhang and Huang 2006; Zhang 2008) is implemented to study the extremal dependence structure among multiple risky financial asset classes through the proxy of the time series of several financial security indices.

2. Preliminaries

2.1. Methodology

The theoretical background and statistical properties of the multivariate EVT framework, which is a combination of max-stable processes and GARCH processes, are well documented in (Smith and Weissman 1996; Smith 2003; Zhang and Smith 2004; Zhang and Huang 2006). (Zhang 2008) develops a new test statistic, namely, the quotient statistic for the hypothesis test of tail dependence/independence. The author denote this new test as the gamma test, because the asymptotic distribution of the quotient statistic is a gamma distribution. As an application of the multivariate EVT model, the quotient statistic, as well as the gamma asymptotical distribution, (Zhang and Huang 2006) applies the M4 process and the associated testing techniques to study the tail dependence structure of three major U.S. stock indices, namely, DJIA, S&P 500, and NASDAQ. These new findings have laid the theoretical foundation for this study. The main objective of this work is to apply these new theories to hypothesis testing and characterization of the extremal dependence structures underlying a broader range of financial asset classes. The similar data processing strategies, testing procedure, and tail dependence measure as documented in (Zhang and Huang 2006) are implemented to accomplish the research objectives in this study.

Specifically, the modeling steps employed in this study are outlined as follows.

In the first step, the data set, which is chosen to be certain financial index time series, is fed into a Generalized Autoregressive Conditional Heteroscedasticity, or GARCH (Bollerslev 1986) model to filter out the volatilities in these time series. After this GARCH fitting step, the heteroscedasticity, which is usually associated with financial time series, is mitigated, and the resultant financial time series become stationary and are approximately i.i.d., thereby far more amenable to the subsequent EVT estimation (McNeil and Frey 2000; Engle 2002; Dias and Embrechts 2003). The standardized time series are also called pseudo-observations (Zhang and Huang 2006). According to (Poon, Rockinger et al. 2004; Zhang and Huang 2006), the EVT analysis results are not sensitive to the parameter choice of the GARCH model.

The exceedences within the standardized time series that are over a specific threshold are then fit to an extreme value distribution. The only two limit distributions of the exceedances over certain thresholds have been shown to be either Generalized Pareto Distribution (GPD), or Generalized Extreme Value distribution (GEV). The connection between these two distributions is established by (Pickands 1975). In this study, GEV is chosen as the distribution to fit the extremal values, since it is a more general approach. GEV is a combination of three types (Gumbel, Frechet, and Weibull) of extreme value distributions, and takes the following parametric form:

$$H(x) = \exp\left(-\left(1 + \xi \frac{x-\mu}{\psi} \right)_{+}^{-1/\xi} \right) \tag{1}$$

where μ is the location parameter, ψ >0 is the scale parameter, and ξ is the shape parameter. The detailed descriptions of the GEV fitting process and the associated W-statistic, and Z-statistic are discussed in (Smith 2003). One point worth noting is that the GEV fitting process is applied to the positive and negative exceedance series, respectively. This enables the model to distinguish the tail dependence during both the financial growing and distressing periods.

With the estimated parameters from the GEV fitting, the transformation formula similar to those proposed in (Coles and Tawn 1994) are employed to convert the pseudo-observations to the unit Frechet scale, which is required by the subsequent hypothesis testing procedure. Values below the specific threshold are transformed based on their ranks.

After the data transformation step, we can employ hypothesis test to explore whether or not certain tail dependence structure is supported by the data. In this study, the quotient test statistic and the corresponding gamma-test as presented in (Zhang 2008) are implemented to determine the tail dependence structures underlying the selected financial time series. According to (Zhang 2008), the quotient test statistic takes the following form:

$$q_{u,n} = \frac{\max_{i \le n}\{(u+W_i)/(u+V_i)\} + \max_{i \le n}\{(u+V_i)/(u+W_i)\} - 2}{\max_{i \le n}\{(u+W_i)/(u+V_i)\} \times \max_{i \le n}\{(u+V_i)/(u+W_i)\} - 1} \qquad (2)$$

where u is the threshold value, W_i and V_i are exceedance values. The asymptotical distribution of $q_{u,n}$ is proven to follow $\Gamma\left(2, 1-e^{-1/u}\right)$(Zhang 2008). The corresponding hypothesis test is set to be:

H_0: X and Y are tail independent.

When $nq_{u,n} > \xi_\alpha$, H_0 is rejected in favor of H_1, where ξ_α is the upper α–th percentile of the $\Gamma\left(2, 1-e^{-1/u}\right)$ distribution. In this hypothesis test, the local window enumeration scheme documented in (Zhang and Huang 2006) is adopted. The gamma test is applied to each local window, which comprises 500 consecutive observations, and the local window will slide from the beginning to end of each time series to expose the time variation of the tail dependence.

Besides the hypothesis test, the so-called tail dependence index:

$$\lambda = \lim_{u \to x_F} P\left(Y > u \middle| X > u\right) \qquad (3)$$

where $x_F = \sup\{x \in R : P(X \le x) < 1\}$ as studied in (Ledford and Tawn 2003; Schalter and Tawn 2003; Zhang and Smith 2004; Zhang and Huang 2006) is also implemented in this work to measure the degree of tail dependence among different financial markets.

2.2. Data

As mentioned in the introduction section, one of the significances of this study is to extend the financial asset classes under consideration beyond the equity assets into other asset categories, such as fixed income assets and commodity assets. The time series of multiple financial security indices will be employed as the proxy for these asset classes to explore the tail dependence structures embedded in them.

Specifically, for the equity market, we study the tail dependence structures underlying the indices of Financial Times index (FTSE 100), Cotation Assistee en Continu (CAC 40), and Deutscher Aktien IndeX (DAX), rather than the three U.S. indices (i.e. DJIA, S&P 500, and NASDAQ), which have been studied by a number of the precious works (Zhang 2005; Zhang and Huang 2006). The time horizon chosen for these equity indices ranges from November 26, 1990 to October 20, 2008, which is the maximum common time horizon for these three indices. In an effort to test the tail dependence structure across different financial asset classes, an equity index and a fixed income index are paired. The S&P 500 index and the yield on the U.S. 10-year Treasury Note are used as the proxies for these two markets, respectively. Similarly, the maximum common time horizon for these two indices, which ranges from January 2, 1962 to October 20, 2008, is chosen for these two time series. As another test on the cross asset class tail dependence structure, the DJIA index (used to proxy the equity market) and the Dow Jones AIG Commodity index (used to proxy the commodity market) are paired for the tail dependence test. Based on the same rationale, January 3, 1991 to October 20, 2008 is determined to be the time horizon for these two time series.

For all of the security indices mentioned above, daily price data are retrieved from Yahoo! Finance for subsequent analysis.

3. Results

3.1. Data Processing and Transformation

Before the actual tail dependence modeling and testing step, the daily price data are first converted to logarithmic returns, due to the statistically appealing properties associated with the return series (Cochrane 2005; Tsay 2005). As an illustration, the negative logarithmic returns of the three selected equity indices are plotted in Fig. 1. From this figure, we are able to identify extremal return observations, jumps in returns, as well as clustering in volatility from all three equity index return series.

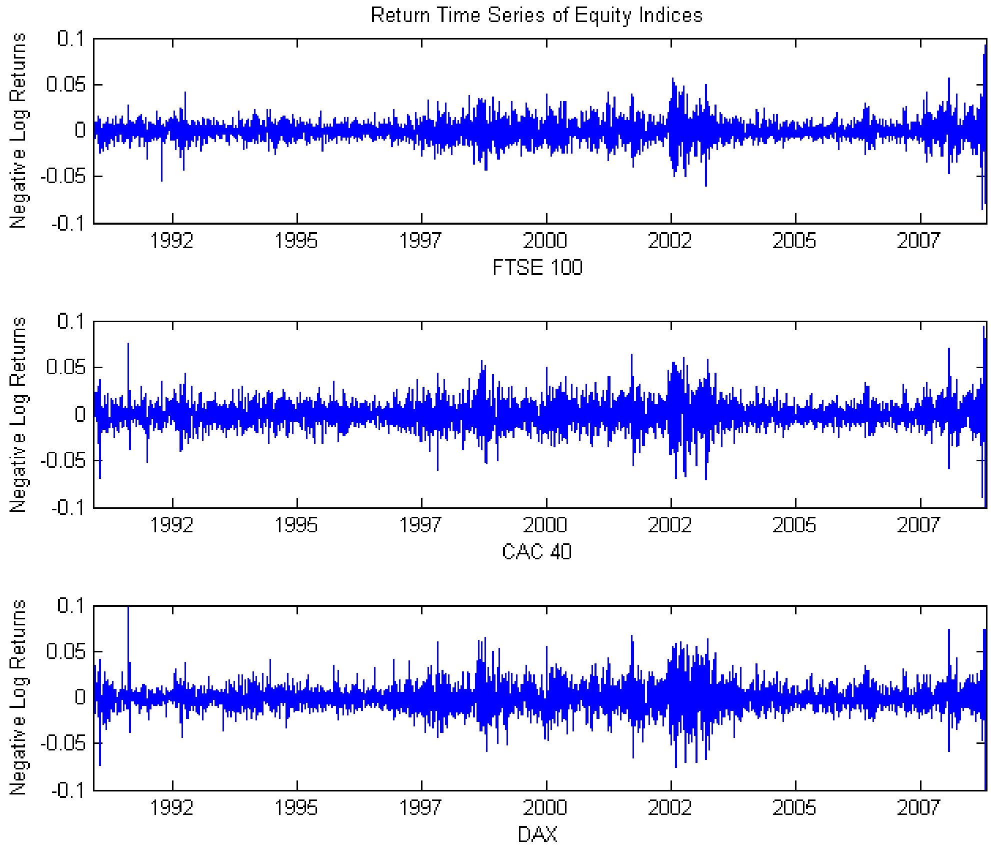

Figure 1: Negative logarithmic return time series of the three equity indices: FTSE 100, CAC 40, and DAX. Daily returns ranging from November 26, 1990 to October 20, 2008 are plotted.

As indicated in (Embrechts, Kluppelberg et al. 1999), the EVT modeling of the tails of a distribution requires the observations to be approximately identically independent distributed (i.i.d.). However, most financial return series exhibit certain degrees of autocorrelation, and more importantly, heteroskedasticity. In order to satisfy the i.i.d. requirement of the EVT modeling, the return series are filtered by a GARCH(p,q) model. The residuals and volatilities embedded in these raw time series are extracted through this GARCH filtering process. According to (Zhang and Huang 2006), different choices of both p and q render similar results. (Poon, Rockinger et al. 2004) also states that the tail dependence results are not sensitive to the choices of the volatility filters. As such, the results presented in this study are all based on a GARCH(1,1) model fitting, which is in consistency with the results reported in (Zhang and Huang 2006). The GARCH(1,1) filtered conditional standard deviations present in all three equity indices are shown in Fig. 2.

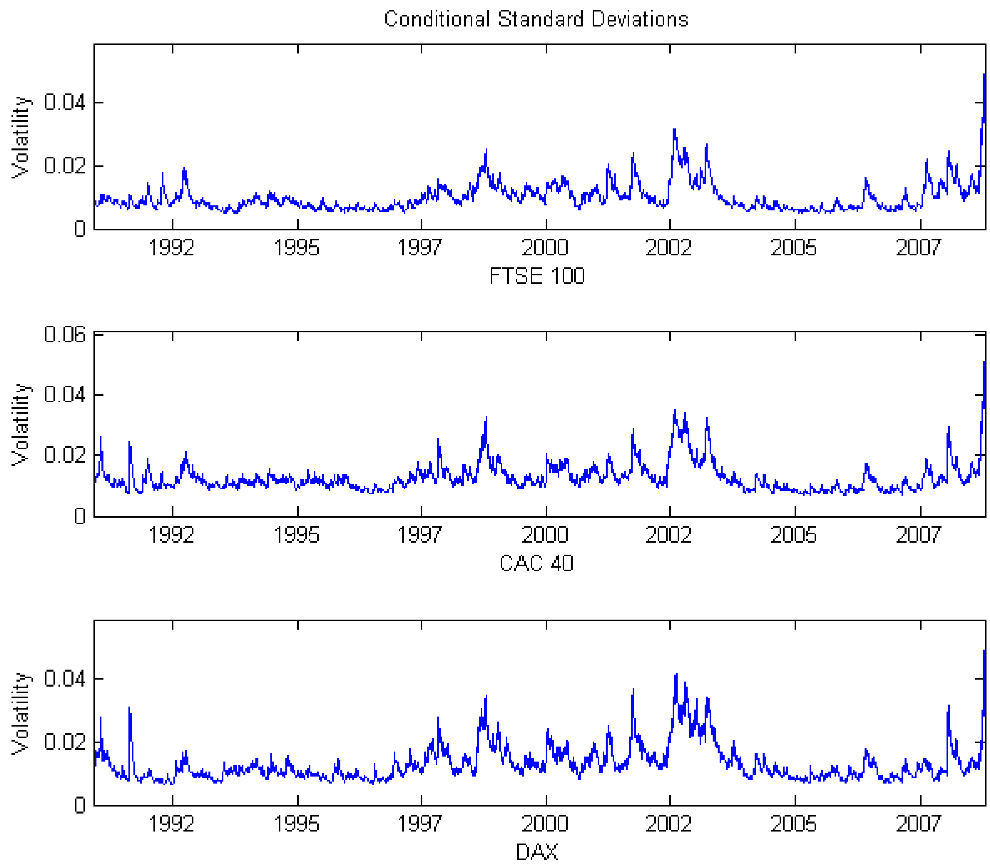

Figure 2: The conditional standard deviation time series corresponding to FTSE 100, CAC 40, and DAX equity indices filtered from a GARCH(1,1) model.

From Fig. 2, it becomes obvious that all return time series demonstrate variations in volatility to certain degree. The conditional standard deviation series remind us of several familiar time spots when the financial markets encounter excess volatility, such as the 1997-1998 Russian credit crisis and Asian financial crisis, and the 2001-2002 tech-bubble period. Additionally, all three time series in Fig. 2 peak at the end period, which put the intensity of the financial turmoil that we are experiencing right now into a historical perspective. The conditional standard deviation series are employed subsequently to standardize the raw return series. The devolatilized return series are also called pseudo-observations (Zhang and Huang 2006), and they have drawn attentions in a large number of financial data analysis, e.g. (McNeil and Frey 2000; Engle 2002; Dias and Embrechts 2003), to name a few. As will be shown shortly (Figs. 4, 5, and 6) with the unit Frechet transformation plots, one can immediately tell that the devolatilized time series are more stationary in comparison to their original counterparts, but the GARCH fitting process does not eliminate the persistency of the extremal jumps present in the return series. This result is consistent with that reported in (Poon, Rockinger et al. 2004) that although heteroscedasticity is a major source of tail dependence, it cannot explain all the market co-movement on its own.

Having filtered the volatility from each return series, the standardized return series (pseudo-observations) are approximately i.i.d. As such, the next task is to fit the exceedances data within the pseudo-observation series that are over a specified threshold to a generalized extreme value (GEV) distribution. The parametric formula of the GEV distribution as given in Eqn. (1) is employed in a maximum likelihood method to estimate the three parameters, namely, the location parameter μ, the scale parameter ψ, and the shape parameter ξ, in the GEV distribution. As mentioned in the methodology section, the GEV fitting is applied to both the positive and negative pseudo-observation time series, separately. The estimated parameters from the GEV fitting are summarized in Tables 1 and 2 for negative and positive return series, respectively.

Table 1
Estimations of parameters from GEV fitting of standardized negative return series.

Indices	N_u	μ **(CI)**	ψ **(CI)**	ξ **(CI)**
FTSE 100	459	1.4853	0.2850	0.3979
		[1.4525, 1.5180]	[0.2572, 0.3159]	[0.2757, 0.5201]
CAC 40	465	1.5122	0.2836	0.3385
		[1.4813, 1.5431]	[0.2581, 0.3117]	[0.2369, 0.4402]
DAX	467	1.4638	0.2584	0.5064
		[1.4350, 1.4926]	[0.2323, 0.2873]	[0.3887, 0.6241]

N_u is the number of exceedances used in the GEV fitting, and the threshold is chosen as $u = 1.2$.

Table 2
Estimations of parameters from GEV fitting of standardized positive return series.

Indices	N_u	μ **(CI)**	ψ **(CI)**	ξ **(CI)**
FTSE 100	497	1.4360	0.2190	0.3695
		[1.4128, 1.4591]	[0.1995, 0.2404]	[0.2686, 0.4705]
CAC 40	471	1.4367	0.2271	0.3712
		[1.4118, 1.4616]	[0.2061, 0.2501]	[0.2638, 0.4785]
DAX	473	1.4245	0.2222	0.3759
		[1.3995, 1.4496]	[0.2011, 0.2455]	[0.2582, 0.4936]

N_u is the number of exceedances used in the GEV fitting, and the threshold is chosen as $u = 1.2$.

In order to provide an intuitive assessment of the GEV fitting, the empirical Cumulative Distribution Function (CDF) of the positive tail exceedances within the pseudo-observation series of the CAC 40 equity index along with the CDF fitted by the GEV are illustrated in Fig. 3. From this figure, we can tell that the fitted distribution closely follows the exceedance data.

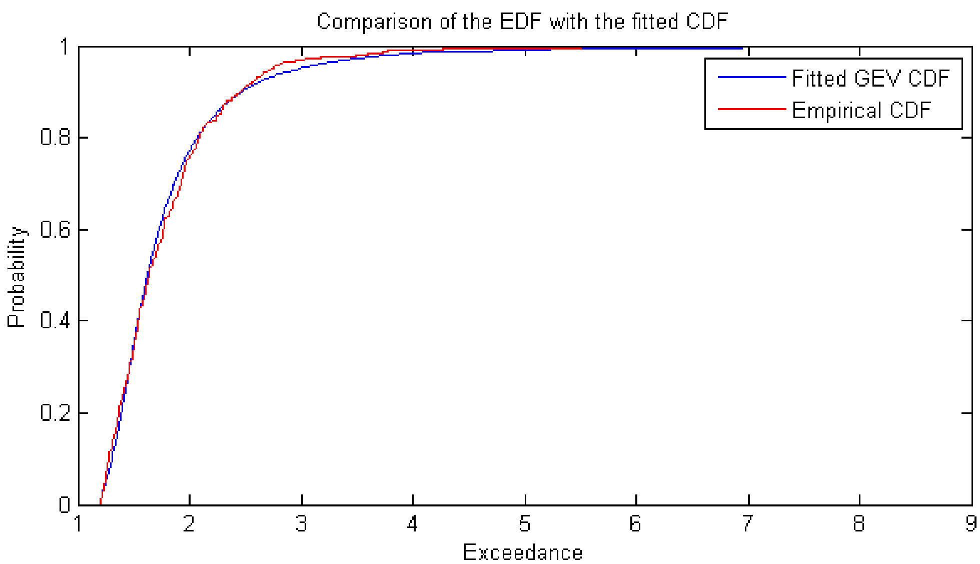

Figure 3: Comparison of the empirical CDF of the positive tail exceedances within the pseudo-observation series corresponding to the CAC 40 equity index with the CDF fitted for the GEV distribution.

Theoretically, the pseudo-observation series can be transformed to any distribution, as long as the distribution function is continuous and strictly increasing. However, the hypothesis testing statistics and the corresponding asymptotic distribution employed in this study require the data to be under the Frechet scale. Hence, without loss of generality, in this study, we transform the pseudo-observation series by the inverse of the unit Frechet distribution function based on the parameters estimated from the previous GEV fitting step. As mentioned before, the positive and negative pseudo-observation series are fitted into GEV distributions separately. Therefore, they are transformed into Frechet distribution separately, as well. The parameters used for the transformation come from the different sets of parameter estimations reported in Tables 1 and 2. With these estimated parameters, the formula similar to those reported in (Coles and Tawn 1994) are employed in this study for transformation. As an illustration, the standardized logarithmic return series, together with the transformed positive and negative series under the unit Frechet scale, are presented in Figs. 4-6 for the FTSE 100, CAC 40, and DAX equity indices, respectively.

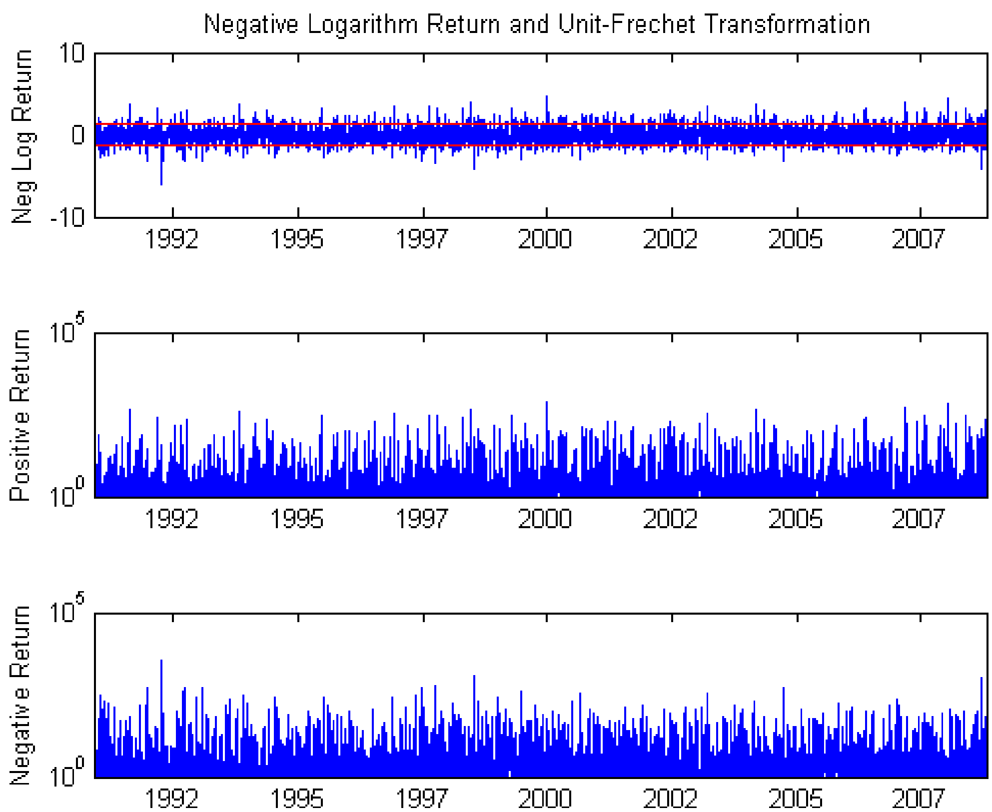

Figure 4: Standardized FTSE 100 negative logarithmic return series and the unit Frechet transformation of both the positive and negative return series.

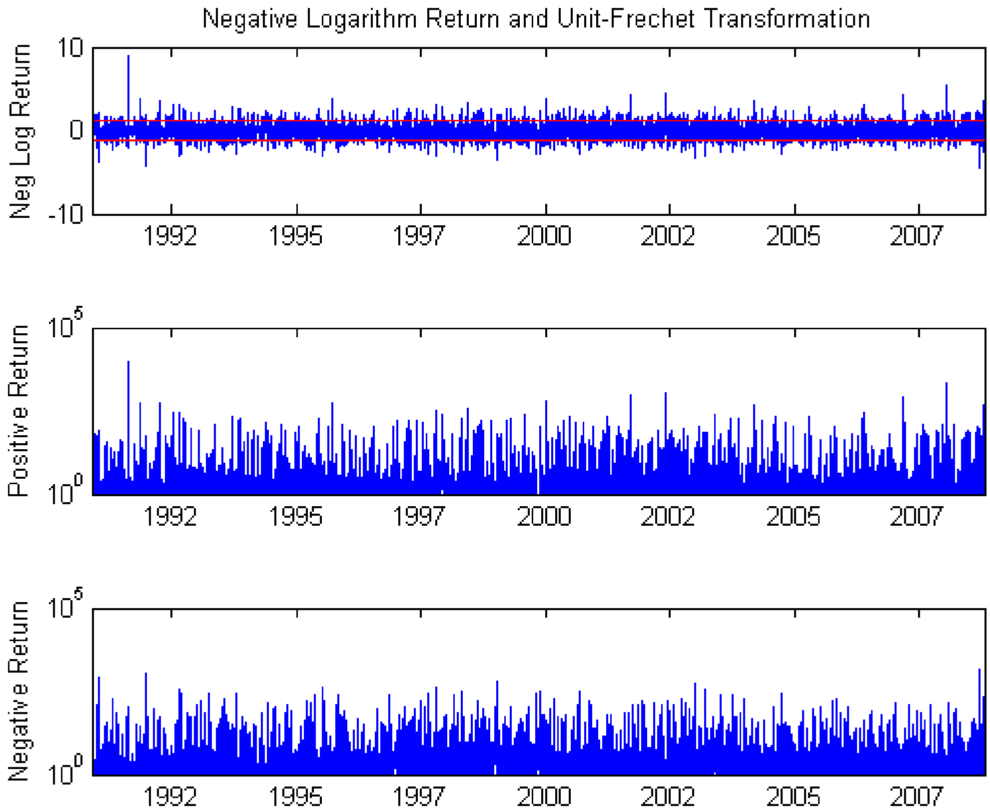

Figure 5: Standardized CAC 40 negative logarithmic return series and the unit Frechet transformation of both its positive and negative return series.

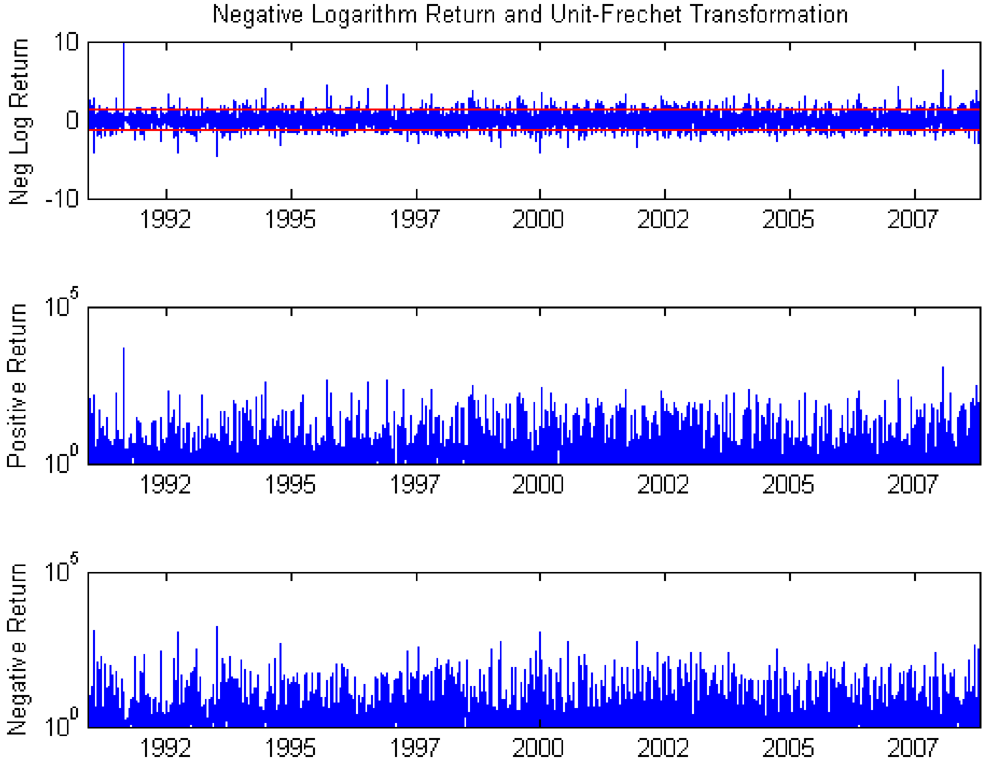

Figure 6: Standardized DAX negative logarithmic return series and the unit Frechet transformation of both its positive and negative return series.

Figs. 4-6 manifest that the unit Frechet transformation is a monotonic transformation of the exceedances, in the sense that the exceedances within all the return time series retain their relative magnitudes before and after the transformation. Direct observation of the series also makes it clear that among these three indices, the CAC 40 index looks more "similar" with the DAX index than with the FTSE 100 index, in the sense that the exceedances with high magnitudes in both the positive and negative constituent series appear at the same time spots on the CAC 40 and DAX indices. Whereas, such pattern is not apparent when we pair FTSE 100 with either CAC 40 or DAX indices. Hence, we have reason to conjecture that the CAC-DAX index pair will have higher extremal dependence than both the FTSE-CAC and FTSE-DAX pairs. This intuition is further proven through the hypothesis test and the computation of the extremal dependence index (λ) presented in the later sections.

Following the same data processing and transformation procedure laid down for the three selected equity indices, the same steps are carried out on all the time series used in both the equity-fixed income and the equity-commodity tail dependence tests. In Table 3 and 4, the GEV fitting parameter estimations are reported for the time series used in the cross asset class tail dependence tests. The pseudo-observation time series as well as their unit-Frechet transformation plots are omitted because of their similarity to the results exhibited in Figs. 4-6.

Table 3
Estimations of parameters from GEV fitting of standardized negative return series.

Indices	N_u	μ **(CI)**	ψ **(CI)**	ξ **(CI)**
S&P 500	1214	1.4495	0.2562	0.5296
		[1.4316, 1.4674]	[0.2395, 0.2740]	[0.4540, 0.6053]
10-Year Note	1073	1.4758	0.2874	0.5504
		[1.4543, 1.4973]	[0.2671, 0.3091]	[0.4672, 0.6335]
DJIA	453	1.4401	0.2619	0.6140
		[1.4098, 1.4704]	[0.2330, 0.2944]	[0.4816, 0.7464]
DJ-AIG	459	1.4767	0.2752	0.4546
		[1.4458, 1.5076]	[0.2480, 0.3054]	[0.3380, 0. 5712]

N_u is the number of exceedances used in the GEV fitting, and the threshold is chosen as $u = 1.2$.

Table 4
Estimations of parameters from GEV fitting of standardized positive return series.

Indices	N_u	μ **(CI)**	ψ **(CI)**	ξ **(CI)**
S&P 500	1228	1.4563	0.2478	0.4056
		[1.4392, 1.4733]	[0.2329, 0.2637]	[0.3347, 0.4766]
10-Year Note	1161	1.4840	0.2943	0.4997
		[1.4626, 1.5054]	[0.2747, 0.3153]	[0.4186, 0.5809]
DJIA	490	1.4497	0.2420	0.3679
		[1.4227, 1.4767]	[0.2193, 0.2670]	[0.2498, 0.4861]
DJ-AIG	476	1.4544	0.2442	0.4427
		[1.4276, 1.4812]	[0.2207, 0.2701]	[0.3306, 0.5548]

N_u is the number of exceedances used in the GEV fitting, and the threshold is chosen as $u = 1.2$.

3.2. Hypothesis Testing

After all pseudo-observation time series are transformed to the unit-Frechet distribution, we can employ hypothesis test to determine whether certain tail-dependence structure is supported by the data. In this study, the quotient test statistics and the corresponding gamma-test as reported in (Zhang 2008) are implemented for the hypothesis testing. According to (Zhang 2008), the hypothesis test is designed to be:

H_0: X and Y are tail independent

H_1: X and Y are tail dependent

When $nq_{u,n} > \xi_\alpha$ ($q_{u,n}$ is the quotient test statistics given in Eqn. (2)), H_0 is rejected in favor of H_1, where ξ_α is the upper α –th percentile of the gamma(2, $1-e^{-1/u}$) distribution. When H_0 is rejected, $q_{u,n}$, the quotient statistics becomes a tail dependence measure. Similar as the study presented in (Zhang and Huang 2006), we use subsets of the pseudo-observation data to calculate the Type I error. In this study, 500 consecutive data points are selected from all the financial time series, and a full enumeration local window scheme as that reported in (Zhang and Huang 2006) is employed as well. In each test, we use the 95-th percentiles of the data as the critical value, and the test significance level is chosen as $\alpha = 0.05$. The testing results are summarized in Table 5 for the three equity indices selected in this study. In this table, Index Pairs indicate the two pseudo-observation time series paired for the hypothesis test, which play the role of the X and Y series in Eqn. (3). For instance, (FTSEn, CACp) means that the test results are reported by calculating the quotient statistics based on the negative return series of the FTSE 100 index and the positive return series of the CAC 40 index. Other testing pairs are defined similarly. In Table 5, Rejection Rate represents the percentage of rejecting $H_{0,}$ when we slide the local window along the entire return series.

Table 5
The hypothesis test results for the extremal dependence structure underlying the three selected equity indices.

Index Pairs	Rejection Rate
(FTSEn, FTSEp)	0.033
(CACn, CACp)	0
(FTSEp, CACp)	0.4811
(FTSEn, CACn)	0.3141
(FTSEn, CACp)	0
(FTSEp, CACn)	0
(DAXn, DAXp)	0.1074
(FTSEp, DAXp)	0.5302
(FTSEn, DAXn)	0.3084
(FTSEn, DAXp)	0.011
(FTSEp, DAXn)	0.0588
(CACp, DAXp)	0.6153
(CACn, DAXn)	0.4015
(CACn, DAXp)	0
(CACp, DAXn)	0.0428

In this discussion, we define the return pair consisting of the time series with the same sign (i.e. positive v.s. positive or negative v.s. negative) to be same-sign-pair and the pair including the return series with the opposite signs (i.e. positive v.s. negative or negative v.s. positive) to be opposite-sign-pair. The hypothesis test results summarized in Table 5 suggest that the degree of extremal co-movements in the same-sign-pair comprising the time series from different equity indices (e.g. FTSEp-CACp or CACn-DAXn), are in general high, as indicated by the high rejection rates for the corresponding return pairs. A larger rejection rate means that the quotient test is rejected over a bigger portion along the corresponding time series. On the contrary, the opposite-sign-pair tends to move independently in the tails no matter whether the two constituent return series come from the same index or from different indices. This conclusion can be drawn from the small rejection rates reported in Table 5 for the corresponding opposite-sign-pairs. Except for the positive-negative return pair with its constituents both coming from the DAX index, the rejection rates for all other opposite-sign-pair are smaller in magnitudes when compared with the rejection rates for the same-sign-pair. The test results for several opposite-sign-pairs are even zero, meaning nowhere on the time series exhibits tail dependence between the two constituent return series. Detailed observations of the results reported in Table 5 also manifest that the degree of extremal dependence between the CAC 40 and DAX indices is higher than that either between FTSE 100 and CAC 40 or between FTSE 100 and DAX, since the rejection rates between CAC 40 and DAX indices are higher than those of other index pairs. This result is consistent with our conjecture by looking at the pseudo-observation time series illustrated in Figs. 4-6.

Following the same hypothesis testing procedure, the extremal dependence structures underlying the equity-fixed income market pair, as well as the equity-commodity market pair are exposed. The results are summarized in Table 6 and 7, respectively.

Table 6
The hypothesis test results for the extremal dependence structure underlying the equity and fixed income markets.

Index Pairs	Rejection Rate
(SP500n, SP500p)	0.1278
(10YEARn, 10YEARp)	0.0602
(SP500p, 10YEARp)	0.0686
(SP500n, 10YEARn)	0.0744
(SP500n, 10YEARp)	0.1841
(SP500p, 10YEARn)	0.0939

Table 7
The hypothesis test results for the extremal dependence structure underlying the equity and commodity markets.

Index Pairs	Rejection Rate
(DJIAn, DJIAp)	0.1111
(DJAIGn, DJAIGp)	0
(DJIAp, DJAIGp)	0.0579
(DJIAn, DJAIGn)	0.0045
(DJIAn, DJAIGp)	0.0961
(DJIAp, DJAIGn)	0.0642

The hypothesis testing results presented in Tables 6-7 demonstrate the following tail dependence structures across different asset classes. The results summarized in Table 6 expose the extremal dependence structure between the S&P 500 index, which is used as a proxy for the equity market, and the 10-Year Treasury Note yield, which is used as a proxy for the fixed income market. As opposed to the tail dependence between two equity indices, as the results reported in Table 5, the tail dependence between the equity and the fixed income return series in general is low. However, the rejection rates for the opposite-sign-pairs are higher than those for the same-sign-pairs, which means that the extremal returns of the opposite sign have a higher co-movement tendency between these two markets. This different tail dependence structure can possibly be explained by the "fly to quality" effect that has long been observed between the equity and the fixed income markets. This conjecture can be further supported by the fact that the rejection rate for the (SP500n, 10YEARp) pair is the highest among the rejection rates for all pairs for these two markets. This means that when the equity market experiences extremal negative returns, the 10-Year Treasury Note market can expect extremal positive returns with relatively high probabilities. The results presented in Table 7 group the tail dependence structure between the equity and the commodity markets and that underlying the equity and the fixed income markets into the same camp. The tail dependence between the equity market and the commodity market is similarly low in general. Although not as significant as the structure exhibited between the equity and the fixed-income markets, the opposite-sign-pairs between the equity and the commodity markets also demonstrate higher tail dependence than the same-sign-pairs do.

3.3. Extremal Dependence Index

As a direct measure to quantify the tail dependence, the tail dependence index as studied in (Ledford and Tawn 2003; Schalter and Tawn 2003; Zhang and Smith 2004; Zhang and Huang 2006) is implemented in this section. The formula of the tail dependence index is given in Eqn. (3), and the index computations are summarized in Table 8.

Table 8
The extremal dependence index for different financial asset classes.

Index Pairs	M.E.D.I	Std. Dev.
(FTSEp, CACp)	0.6021	0.1137
(FTSEn, CACn)	0.5289	0.1390
(FTSEn, CACp)	0	0
(FTSEp, CACn)	0	0
(FTSEp, DAXp)	0.4880	0.1114
(FTSEn, DAXn)	0.4277	0.1092
(FTSEn, DAXp)	0.0099	0.0139
(FTSEp, DAXn)	0.0056	0.0097
(CACp, DAXp)	0.5645	0.1185
(CACn, DAXn)	0.5189	0.1400
(CACn, DAXp)	0.0042	0.0073
(CACp, DAXn)	0.0097	0.0131
(SP500p, 10YEARp)	0.0973	0.0955
(SP500n, 10YEARn)	0.0786	0.0953
(SP500n, 10YEARp)	0.2007	0.0907
(SP500p, 10YEARn)	0.2220	0.1115
(DJIAp, DJAIGp)	0.0996	0.0544
(DJIAn, DJAIGn)	0.1245	0.0424
(DJIAn, DJAIGp)	0.1146	0.0359
(DJIAp, DJAIGn)	0.1107	0.0431

In this table, M.E.D.I. stands for the average empirical estimation of the extremal dependence index. Std. Dev. represents the sample standard deviation of the extremal dependence index seires.

From Table 8, we can see that the tail dependence index results are consistent with the hypothesis testing results reported in Tables 5-7. The tail dependence index of the same-sign-pair within the equity market is the highest among all return pairs. The opposite-sign-pair from the equity market exhibit the least extremal dependence, as reflected by the close-to-zero M.E.D.I. The opposite-sign-pair for the equity and the fixed income markets and the equity and the commodity markets exhibits higher M.E.D.I. than the same-sign-pair coming from these markets. This phenomenon is more apparent between the equity and the fixed-income markets. In this sense, the gamma test is indeed a tail dependence measure (Zhang and Huang 2006).

Besides the average empirical tail dependence index reported in Table 8, the time series of the tail dependence index are also reported in Figs. 7-9 based on the full local window enumeration scheme, in order to further expose the evolution of the tail dependence structures underlying different financial markets.

From Fig. 7, we see that the tail dependence for the same-sign-pairs within the equity market is high, as reflected by the high magnitude of λ. This is consistent with the hypothesis testing results reported in the previous section. Moreover, the time series tell us more about the evolution of λ. As illustrated from all three subplots in Fig. 7, all λ series apparently exhibit upward trends. Since λ is a tail dependence measure, this upward trend means that the degree of tail dependence underlying the corresponding equity markets increases during the testing period. The result plotted in Fig. 7 is consistent with the tail dependence testing results for the European equity markets reported in (Poon, Rockinger et al. 2004), but in a more intuitive fashion. According to (Poon, Rockinger et al. 2004), the presence of tail dependence will render the traditional way of risk evaluation and management inefficient, if not invalid. If a model fails to consider the tail dependence underlying certain financial markets when they are actually tail dependent, it will probably overlook risk factors, hence underestimates the associated risk. As the degree of tail dependence increases, which is represented by the upward-trended λ series, the degree of erroneous risk evaluation and management also increases accordingly. As the misestimating/mismanaging accumulates, may it lead to a new financial market crash?

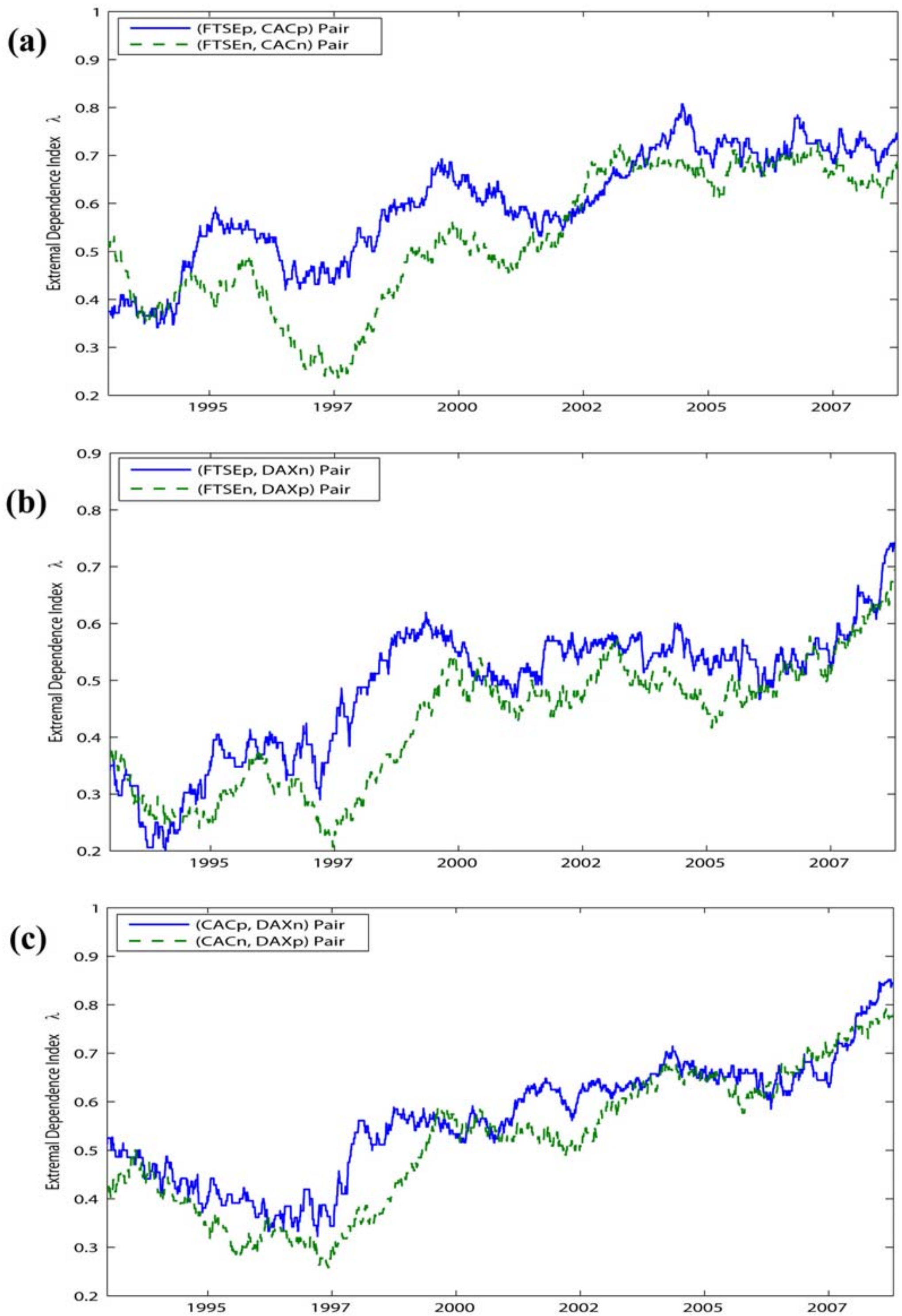

Figure 7: Time series of the tail dependence index λ for the same-sign-pair between (a) FTSE 100 and CAC 40 indices; (b) FTSE 100 and DAX indices; (c) CAC 40 and DAX indices.

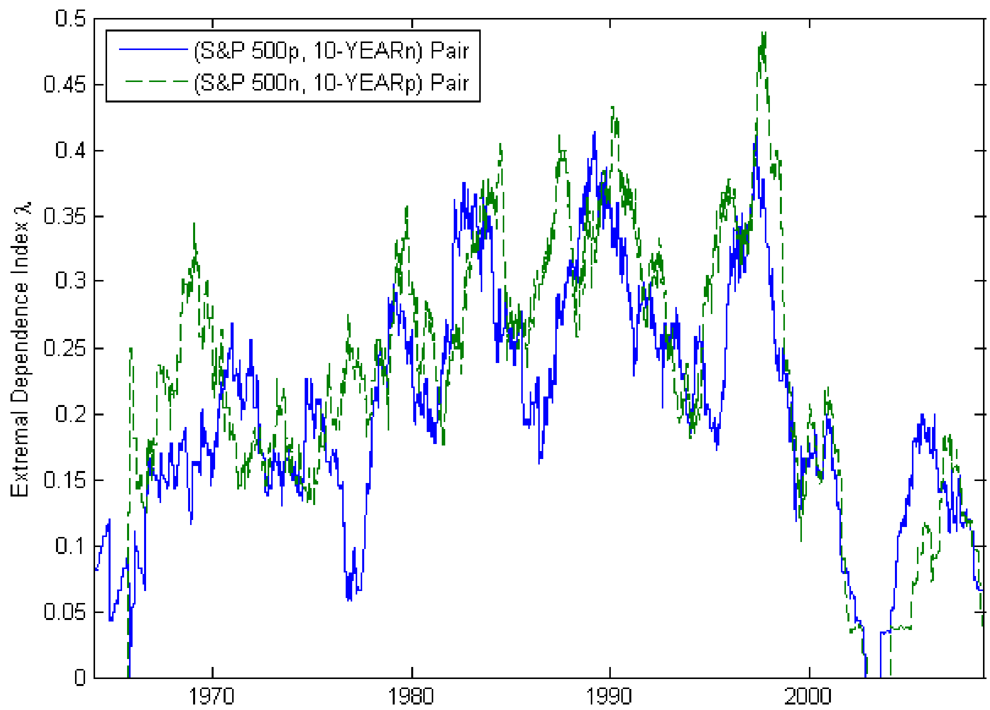

Figure 8: Time series of the tail dependence index λ for the opposite-sign-pair between the S&P 500 equity index and the 10-Year Treasury Note yield.

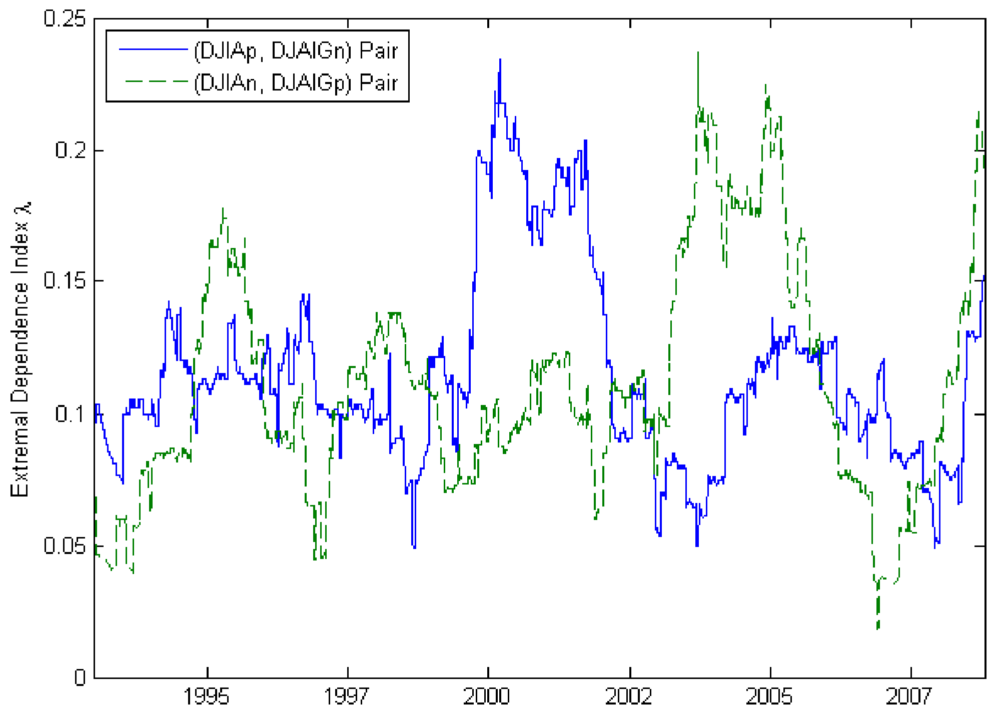

Figure 9: Time series of the tail dependence index λ for the opposite-sign-pair between the DJIA equity and the DJAIG commodity indices.

From Fig. 8, the time series of the tail dependence index for the opposite-sign-pair between the equity and the fixed-income markets exhibits a different pattern from the series in Fig. 7. Prior to 1997, the tail dependence index between these two markets has a clear upward trend, meaning the degree of tail dependence increases between these two markets during this period. After 1998, the tail dependence index experiences sharp drop down to zero, and then continue with an oscillatory increasing pattern during the recent several years. Similarly, the tail dependence index for the opposite-sign-pair between the equity and the commodity markets, as plotted in Fig. 9, exhibits yet another different pattern. Both series do not show clear directional trend, rather demonstrate an oscillatory pattern. In addition, as opposed to the results shown in Figs. 7 and 8, where the two constituent series in each figure exhibit high correlations, the two series in Fig. 9 demonstrate a clear lag phenomenon. This pattern is prominent during the 1990 to 2007 period. During this period, it seems that the tail dependence index between the (DJIAn, DJAIGp) pair lags four years behind that for the (DJIAp, DJAIGn) pair. This lag phenomenon may manifest the economic cyclic period underlying these two markets.

The study in this section extends the research presented in the similar studies (Poon, Rockinger et al. 2004; Zhang and Huang 2006), which only focus on the tail dependence structure underlying the equity markets. Both the hypothesis testing and the tail dependence index results show that the tail dependence across different types of financial markets, e.g. the equity market v.s. the fixed-income market and the equity market v.s. the commodity market, demonstrates clear different structures from the structure underlying the same kind of financial market. When we try to apply the extremal dependence results to the practical scenarios as proposed in (Poon, Rockinger et al. 2004), namely, the portfolio choice, the Sharpe Ratio sharpening, the hedging strategy adjustment, the complex option valuation, and the credit risk analysis, we should be aware of the different tail dependence structures underlying different financial markets. Given an investment universe spanned by multiple financial asset classes, the recognition of the difference in tail dependence structures will lead to better informed, thus potentially optimal decisions.

4. Conclusion

In this study, a multivariate extreme value framework is implemented, and the Extreme Value Theory (EVT) is employed to characterize the tail dependence structures of various financial asset classes. The EVT framework implemented in this study enables us to research the widespread extremal dependence phenomenon in the financial markets, which has been overlooked in the finance literature (Poon, Rockinger et al. 2004). Such omission may lead to erroneous estimation of market risks. As to the specific multivariate EVT model, a newly developed test-statistic, namely, the quotient statistics, as well as the associated gamma testing procedure are implemented in this study to efficiently explore the tail dependence structures underlying various financial markets.

Another contribution of this study is that it overcomes the limitations in the previous similar research, such as (Zhang 2005; Zhang and Huang 2006) and (Poon, Rockinger et al. 2004), which only deal with the tail dependence within the equity market. Due to such limitation, the above researches keep silent about the tail dependence structure underlying different asset classes, hence is less helpful in addressing the needs of the multi-strategy investment vehicles, whose investment universe easily span beyond the equity world. Besides the equity market, this study expands its scope of researching the extremal dependence structure into other financial markets, such as the fixed-income security market and the commodity market. From both the hypothesis testing and the tail dependence index computation results presented in Section 3, we can see clearly that the tail dependence structures underlying different financial markets are not unanimous. Treating the extremal dependences for various financial markets equally will probably lead to erroneous conclusions and suboptimal investment choices.

The multivariate EVT framework, the statistical testing method, as well as the tail dependence measure implemented in this work can serve as a useful tool in exploiting the innovative EVT based arbitrage opportunities and risk management strategies within a certain asset class and across different asset classes.

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