

Index Dividend Shock: What Does the Market Imply and How Does Hedging Work?

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December 5, 2008

Abstract

Dividend cuts cluster in severe financial crisis and bring shock to the index dividend. Our empirical test shows that the index options and futures market reflects the expectation of dividend shock correctly. There are very few arbitrage opportunities in regular market with market friction, but some persistent mispricings do exist when the market is in turmoil. Based on the empirical result, we find that a theoretically perfect hedging strategy for index dividend shock is extremely costly and volatile in the real market, and we propose a practical strategy that has similar performance as the perfect hedging strategy but incurs much lower cost and volatility.

1 Introduction

Though various indexes, index options and index futures have been extensively traded for a long period, direct trading on index dividend is a somewhat new practice in the financial market. Index dividend swap existed in the OTC market for only several years, and the index dividend future was launched just recently as a standardized product for index dividend trading. The fast growth of the index dividend trading market calls for more attention to the empirical research in this market.

As we know, a dividend is a payment corporations make to the shareholders. Usually, corporations will retain part of their earnings for reinvestment while paying out the remainder as the dividend. The dividend cut is usually deemed an unfavorable signal of future performance, and some researchers (e.g. Brav, Graham, Harvey and Michaely (2005), Choe(1990), Fama and Babiak (1968), Lintner (1956)) have found that a few corporations try hard to "manage" their reinvestment in order to smooth the dividend payment each year. As a result, the index dividend is expected to be relatively stable and bear very low volatility in the regular market. However, in some severe economic or financial crises when most corporations are unable to maintain their targeted dividend level given the poor earnings, these corporations are forced to cut their dividends simultaneously. The clustering of dividend cuts

in these crises will bring shock to the index dividend and may hit investors badly.

When marketwide crisis is anticipated, investors can hedge their investment in index dividend before corporations declare their dividend cuts. A direct liquidation of all index dividend inventory proves to be painful in a marketwide sell-off, and a direct opposite position in index dividend contracts is difficult to implement in the short term when the previous holding is large relative to the index dividend market. Theoretically, index options and index futures should reflect the market expectation of dividend payment correctly, and they are more liquid in a deeper market. So one potential hedging strategy we propose in this paper is to hedge the index dividend based on put-call parity or future pricing model. The effect of this hedging strategy greatly depends on whether the market price of index options or index futures reflects the index dividend expectation correctly. One of our contributions in this paper is that we test the efficiency of the options and futures market. Our test is based on the no-arbitrage relationship among financial instruments, that is, the put-call parity proposed by Stoll (1969) and the traditional index future pricing model. A few researchers contributed to the put-call parity test, including Bhattacharya (1983), Gould and Galai (1974), Klemkosky and Resnick (1979,1980), Merton (1973) and Nisbet (1992). We focus our test on the SX5E index, and, to our best knowledge, no previous papers

conducted any empirical studies on this important European index. Most research investigated the American market and only several recent papers involved the European market: Capelle-Blancard and Chaudhury (2001) for the French index (CAC40) options market; Mittnik and Rieken (2000a) for the German index (DAX) options market; and Cavallo and Mammola (2000) and Brunettia and Torricelli (2004) for the Italian index (Mib30) options market. Our test is carried out in two steps: first, we will test put-call parity and future pricing model without any market friction to check whether the implied dividends calculated from different instruments are consistent with each other; and second, we will repeat these tests with true market friction to see whether arbitrage opportunities emerge. In the first step, our result is mixed. We found that the implied dividends calculated from different instruments for year 2008 are quite close to each other, but the implied dividends calculated for year 2009 deviate greatly. In the second step, very few arbitrage opportunities appear in our test in the regular market with the existence of market friction. But some arbitrage opportunities do emerge and persist when the market tumbles and investors overreact to some extent.

The test of put-call parity and future pricing model is not the final purpose of this paper but is only the starting point. These empirical results motivate us to reconsider the hedging strategy for the index dividend. We first propose a straightforward and theoretically perfect hedging strategy for

index dividend swap/future, and show that the value of the fully hedged portfolio is very volatile in the real market based on our empirical results. Then, we suggest a practical hedging strategy using the estimation of the index dividend payment schedule and prove that it is a perfect hedging strategy when all the index member companies cut dividends proportionally. When these companies do not cut dividends proportionally, the performance of this hedging strategy depends on the dispersion of index dividend payments. This practical hedging strategy, though not perfect in most cases, is effective and incurs much lower costs.

To sum up, our research contributes to the literature in three aspects. First, as far as we know, no previous research has been conducted in the index dividend trading and hedging field. This paper is dedicated to finding a proper strategy to replicate and hedge the index dividend contract and thus will serve as a starting point for more comprehensive research in this field. Second, to create an effective hedging strategy, we test the efficiency of the options market and futures market for the SX5E index. The SX5E is a very important index in Europe and it has a long history (since 1992), but surprisingly, no previous research ever tested this market. Our work shows that the market is generally efficient with very few arbitrage opportunities under normal conditions, but we do find some persistent market mispricings in market turmoil. Third, we propose a practical hedging strategy and test

its performance using ex post data. The strategy is proven to be very effective and provides strong protection to index dividend investors when large a shock occurs.

In the next section, we briefly describe the financial instruments involved in this paper and summarize the data set we use for empirical tests. The methodologies and models used for testing put-call parity and future pricing models come in section 3, and we present our empirical results afterward. The hedging strategies for index dividend are discussed in detail in section 4 and we conclude in the last section, section 5.

2 Index Market Overview and Data Set

2.1 Index Market: Options, Futures and Dividends

The Dow Jones EURO STOXX 50 (SX5E) Index is a free-floating market capitalization weighted index of 50 European blue-chip stocks from those countries participating in the EMU. It was developed with a base value of 1,000 as of December 31, 1991. It provides a blue-chip representation of supersector leaders in the Eurozone. A large family of index options are created with different maturities and different strikes on the SX5E index. For example, the SEZ8C Y and SEZ8P Y series are European-style call and

put options on SX5E with different strikes. They mature on 12/18/2008 and are traded in Eurex. The option trading period is 9:00-17:30 local time, with the underlying index trading period 9:00-20:00 local time. The index futures market is also very active, and the VGZ8 Index is the SX5E index futures contract which matures on 12/18/2008. It is also traded in Eurex and the trading period is 1:50-16:00 local time.

Compared with these standardized products, the dividend swap contract in the OTC market is much less liquid. It is an important innovation because it allows the investors to gain direct exposure to the index dividend. The payoff of the swap from the long side perspective is equal to the difference of actual total dividend payment (calculated in index points) made by the 50 companies in the SX5E index and the swap strike price during a certain period. These 50 companies pay dividends at different times during the year, but the time value of money is not considered. The formula for the dividend swap payoff is:

$$Payoff_{Div} = \sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij}) - K^{Div} \quad (1)$$

$Div(t_{ij})$: the j th dividend paid out by company i at time t_{ij} during the contract period

K^{Div} : the strike price of the dividend swap or the price of dividend future

M : the total number of member companies in the index; for SX5E, $M=50$

N_i : the total times of dividend payment made by company i during the contract period

In this paper, i is usually used to label the company in the index and j is usually used to label the dividend payment by a certain company. (A company may pay out multiple dividend during the contract period, e.g. , the quarterly dividend or semiannual dividend.)

Recently, the index dividend future contract was launched. The index dividend future has the identical payoff as the index dividend swap but is more actively traded and more liquid. For instance, the DEDZ8 Index is the dividend future contract for 2008, covering the period from 12/27/2007 to 12/19/2008.

2.2 Data Set Description

All the data used in this paper are available in Bloomberg database except the dividend swap price. Since the dividend swap is an OTC market product, we obtained the price data from brokers. At the same time, we used the dividend future price to substitute for the dividend swap price as soon as the dividend future was issued in June 2008. The dividend future price can be easily obtained from Bloomberg database, so after June 2008, all the data we employ are available to the public.

2.2.1 The data synchronicity

Both the test of put-call parity and the test of index future pricing model require input of different instrument prices at the same time. Synchronicity is the most significant issue in empirical testing, and researchers worked hard to guarantee a high level of synchronicity. For instance, Capelle-Blancard and Chaudhury (2001) and Mittnik and Rieken (2000) used a sample in which prices were required to be within 1 min of each other, while Cavallo and Mammola (2000) used intraday prices captured every 15 min.

As we mentioned before, the SX5E index trading period is different from the index options and index futures trading period, so the close price can't be used because the index market closes hours later than the options and futures market. In this paper, we pick the overlapped trading hours of index and options/futures each day and sample the intraday prices every 30 min. This overlapped trading period sampling method provides us with plenty of synchronous trading data.

The intraday data is summarized in Table 1.

Instruments	Intraday Observation	Data Period	Total Observation
Options	17/day	12/27/2007-10/22/2008	3587
Futures	17/day	3/25/2008-9/29/2008	2278
Index	17/day	12/27/2007-10/22/2008	3587

Table 1. Summary of Intraday Data

2.2.2 The risk-free interest rate

The risk-free rate used in put-call parity and the index future pricing model is critical for the test. Most researchers resort to interbank offer rates and adjust the offer rates by borrowing and lending spread. The SX5E index and relevant options and futures are traded in the European market, so we use the Euribor rate as the risk-free interest rate in this paper. When the hedge portfolio is created, we match the maturity of options or futures with the Euribor rate term structure. For example, if we are trying to establish a hedge portfolio with options that are 6 months to maturity, then we use the 6-month Euribor rate as the discount rate. This matching method has proved to be very important. We find that when no matching method is employed, the implied dividend calculated from put-call parity or index future will deviate greatly from the implied dividend calculated from the index dividend swap/future (i.e., the test result is worse). And when the match-

ing method is used, the result is then greatly improved. These phenomena, on the other hand, validate the necessity of using the Euribor rate and the matching method.

As for the borrowing and lending spread, we assume a fixed spread over time. Basically, we assume that the lending rate is 25 bps lower than the Euribor rate and the borrowing rate is 25 bps higher than the Euribor rate. The test can easily be extended to fit other borrowing-lending spread setups. The intraday Euribor rate term structure data usually have no variation, so we use the close price of Euribor rate in our paper.

2.2.3 Dividend

The SX5E index is a price index, so the index dividend is important for both put-call parity test and future pricing model test. Our purpose concerning all the tests presented in this paper is not only to check whether the market is efficient, but also to use the information to design and backtest the index dividend hedging strategies. Our tests are carried out as follows: we first calculate the implied dividend from put-call parity or index future pricing model without considering any market friction, and then we calculate the implied dividend from the index dividend swap/future. If these two implied dividends are close to each other statistically, we claim that the market is efficient. Otherwise, we further test the existence of arbitrage opportuni-

ties with market friction such as bid-ask spread, borrowing and lending rate spread, short selling rebate and other transaction costs.

Previous researchers dealt with dividend a bit differently than we do. They usually assume that the investors have perfect foresight regarding the dividends to be paid and use the ex-post dividend data in their test (e.g., Capelle-Blancard and Chaudhury (2001), Kamara and Miller (1995)). This methodology works well in a regular market in which the index dividend payment estimation is quite stable over the year. However, it does not work in the scenario we care about, that is, in an economic or financial crisis when quite a few companies unexpectedly cut dividend at the same time. In this case, the expected index dividend to be paid out during the year may change drastically over time, and thus the ex-post dividend payment may not always be a good proxy for the market expectation of dividend, especially in the early stages of the crisis. We use the implied dividend calculated from the index dividend swap/future as the proxy, and it is a much better proxy; because the index dividend swap/future trades directly on the index dividend, and thus its price reflects the instantaneous market expectation of index dividend level at any time. Because it checks whether the implied dividend calculated from the put-call parity or future pricing model is consistent with this proxy, theoretically our test will be more reliable.

2.2.4 Transaction cost

Transaction cost plays a critical role in no-arbitrage tests. Most previous studies concluded that in practice arbitrage opportunities are in practice swept away by the transaction costs involved in creating arbitrage strategies. Transaction costs come from the bid-ask spread, borrowing-lending rate spread, short-selling rebate rate and commission fees. Nearly all the previous literature considered at least parts of all these factors. In practice, transaction costs are usually very difficult to measure because of the lack of reliable data. In this paper, we use the market-observed bid-ask spread and assume the borrowing-lending rate spread to be 50 bps. We design several scenarios in which the short-selling rebate and commission fees vary and find that the test result is robust because the first two factors eliminate most arbitrage opportunities.

3 Test Methodologies and Empirical Results

3.1 Implied Dividend Test For PCP

The relationship between European put and call can be summarized as follows:

$$P_t + S_t = C_t + PV(K^S) + PV\left(\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})\right) \quad (2)$$

P_t : the price of European put at time t

S_t : the price of index at time t

C_t : the price of European call at time t

$PV()$: the operator of present value

K^S : the strike price of options

$Div(t_{ij})$: the j th dividend paid out by company i at time t_{ij} during the contract period

Rearranging the equation and compounding both sides to the maturity date T , we find that the total implied future value of dividend reflected in put-call parity is:

$$FV\left(\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})\right) = FV(P_t) + FV(S_t) - FV(C_t) - K^S \quad (3)$$

$FV()$: the operator of future value

The implied dividend amount calculated by equation 3 includes all the dividend that is expected to be paid out during time t and option maturity date T . For example, if we evaluate equation 3 on 01/01/2008 and the options have a maturity date of 12/31/2009, then the implied dividend includes all the dividend that is expected to be paid between 01/01/2008 and 12/31/2009,

that is, the total amount of the two-year dividend. As a result, if the investors want to hedge only the 2009 dividend payment, they need to create two sub-portfolios. The first sub-portfolio is created by options with maturity date at the end of year 2008, and this sub-portfolio can establish a synthetic long or short position on 2008 dividend payments only. The second sub-portfolio is created by options with maturity date at the end of year 2009, and this sub-portfolio can establish a synthetic long or short position on 2008 and 2009 dividends payment together. Thus, for example, if the investors want to long the implied dividend of year 2009 only, they can long the second sub-portfolio and short the first sub-portfolio.

Once the synthetic position on implied dividend is created by put-call parity as in equation 3, the investors simultaneously take an opposite position on the actual dividend to be realized. To see why this is done, let's assume that we long the implied dividend using equation 3. In this case, we short a put option, short index, long a call and set aside the present value of strike to earn the risk-free rate. The total cash flow generated when we create this portfolio is positive, and it is exactly the present value of the implied dividend we will get. However, during the period we hold the portfolio to the maturity date (on the maturity date, the portfolio is worth exactly zero), we need to pay for the actual dividend to be realized because we short the index. As a result, the total payoff for this portfolio is simply the difference between

the implied dividend and the actual dividend to be realized. Of course, we need to adjust for the time value of money properly because the cash inflow and outflow occur at different times.

The first step we need to test is whether the implied dividend calculated from put-call parity in equation 3 is close to the implied dividend calculated from index dividend swap/future when no market friction is considered. We calculate the absolute value of the difference between these two implied dividends in percentage form and monetary form using the following equations:

$$Diff_{percent} = \frac{|Div_{imp}^{PCP} - Div_{imp}^{DivSwap}|}{Div_{imp}^{DivSwap}} \quad (4)$$

$$Diff_{moneta} = |Div_{imp}^{PCP} - Div_{imp}^{DivSwap}| \quad (5)$$

Div_{imp}^{PCP} : the implied dividend calculated from put-call parity

$Div_{imp}^{DivSwap}$: the implied dividend calculated from dividend swap

The quantiles of the absolute difference are reported separately for the 2008 implied dividend and 2009 implied dividends.

The empirical results are summarized in Table 2 and Table 3.

		2008		2009	
Quantile	Percentage	Monetary	Percentage	Monetary	
10%	0.32%	0.51	1.12%	1.62	
30%	0.89%	1.43	4.03%	6.07	
50%	1.45%	2.30	7.10%	10.73	
75%	2.26%	3.59	12.01%	18.29	
90%	3.65%	5.72	18.31%	27.35	
95%	5.23%	8.09	30.98%	33.58	

Table 2. Quantiles of Absolute Value of Difference

		2008		2009	
Total Intraday Obs.		3587		3570	
Total Day Num		211		210	
		Monetary	Percent	Monetary	Percent
Intraday Stdev Mean		1.40	0.89%	2.09	1.44%
Intraday Stdev Median		0.98	0.61%	1.47	0.97%

Table 3. Summary of Implied Dividend Difference

From Table 2, we notice that the 2008 implied dividends calculated from put-call parity and index dividend swap/future are very close to each other statistically. The 90% quantile of the absolute value of difference in percent-

age form is only 3.65%. It is equivalent to say that we are 90% sure that the implied dividend calculated from put-call parity will not deviate more than 3.65% from the implied dividend calculated from index dividend swap/future. The absolute value of difference in monetary form is also very low, only 5.72 compared with the 2008 index dividend level, which remains at 150 to 160 all through the year. However, the 2009 test result is not as good as that of 2008. A 18.31% deviation for the 90% quantile shows that significant difference exists sometimes between the implied dividend calculated from put-call parity and that calculated from index dividend swap/future. We have to emphasize here that the significant divergence of these two implied dividends does not guarantee profitable arbitrage opportunities, because we do not consider any market friction so far. This result indicates only that more noise exists when we try to create a synthetic position in year 2009 index dividend using put-call parity and that we may have more mark-to-market risk if we try to hedge the 2009 index dividend swap/future using put-call parity.

From Table 3, we find some important information about intraday standard deviation of the implied dividend calculated from put-call parity. Though the absolute value of the difference is significantly higher in year 2009, the standard deviation of the implied dividend in year 2009 is close to that in year 2008 in both monetary form and percentage form. This implies that the divergence of these two implied dividends persists for a long time. The

result is illustrated in Figure 1 and Figure 2, where the implied dividends calculated from different instruments for year 2008 and 2009 are plotted.

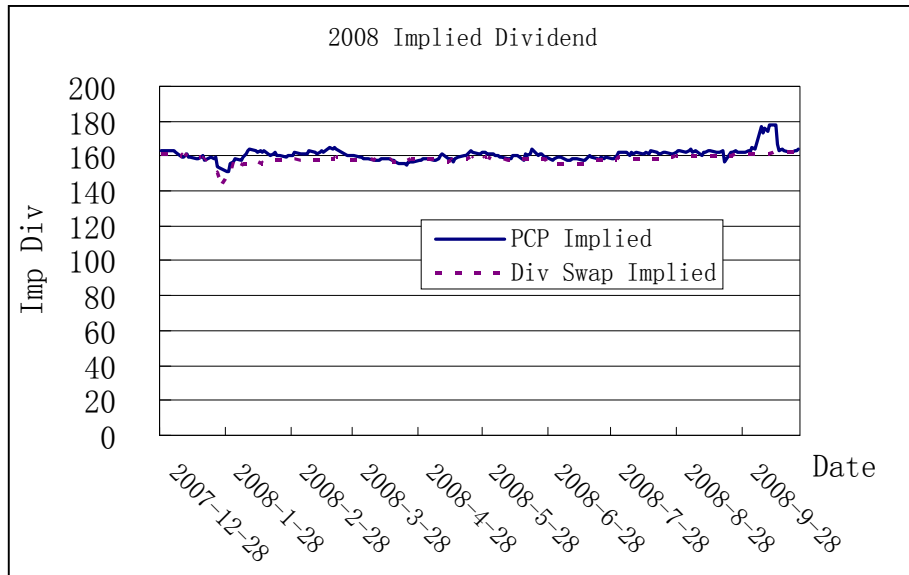


Fig 1. 2008 Implied Dividend Calculated From PCP and Div Swap

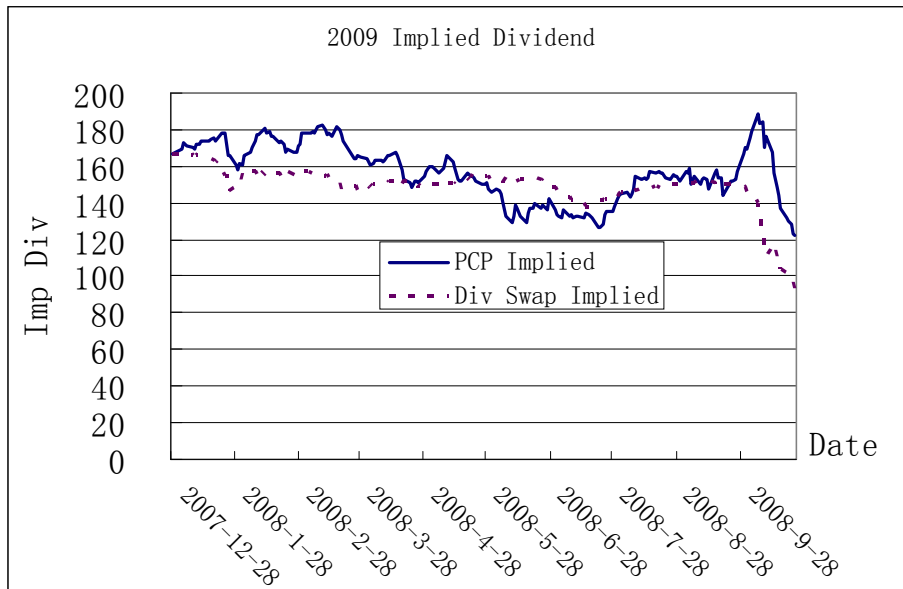


Fig 2. 2009 Implied Dividend Calculated From PCP and Div Swap

3.2 No-Arbitrage Test For PCP

When the implied dividend calculated from put-call parity deviates from the implied dividend calculated from index dividend swap/future, the arbitrage opportunity may appear. To test whether the arbitrage opportunity exists, we need to consider friction such as bid-ask spread, borrowing-lending rate difference, rebate rate on short selling and commission fees.

To create a synthetic long position in the implied dividend (and thus a synthetic short position in dividend to be realized) through put-call parity, we need to long the European call option with strike price K^S at ask price (cash outflow); short the European put option with same strike price and maturity at bid price (cash inflow); short the underlying index at the current market price (cash inflow); and set aside the present value of K^S at the lending rate (cash outflow). The future value of the implied dividend we can get as cash inflow is:

$$FV_L\left(\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})\right) = FV^L(P_{b,t}) + FV^L(S_{b,t}) - FV^L(C_{a,t}) - K^S - FV^L(TC) \quad (6)$$

$FV^L()$: operator of future value, compounding at the lending rate

$P_{b,t}, S_{b,t}$: bid price for put option and index respectively at time t

$C_{a,t}$: ask price for call option at time t

TC : transaction cost including short-selling rebate and commission fee

Similarly, to create a synthetic short position in the implied dividend (and thus a synthetic long position in the dividend to be realized), we need to short the European call option with strike K^S at bid price (cash inflow); long the European put option with the same strike price and maturity at ask price (cash outflow); long the underlying index at the current market price (cash outflow); and borrow the present value of K^S at the borrowing rate (cash inflow). The future value of implied dividend we need to pay as cash outflow is:

$$FV_S\left(\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})\right) = -FV^B(P_{a,t}) - FV^B(S_{a,t}) + FV^B(C_{b,t}) + K^S - FV^B(TC) \quad (7)$$

$FV^B()$: operator of future value, compounding at the borrowing rate

$P_{a,t}, S_{a,t}$: ask price for put option and index respectively at time t

$C_{b,t}$: bid price for call option at time t

TC : transaction cost including only the commission fee

The arbitrage opportunity exists if the following inequality is broken:

$$FV_L\left(\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})\right) < FV_{DivSwap}\left(\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})\right) \quad (8)$$

$$FV_S\left(\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})\right) < -FV_{DivSwap}\left(\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})\right) \quad (9)$$

Intuitively, if equation 8 above is not true, we can long the implied dividend through put-call parity as in equation 6 and short the implied dividend through the index dividend swap/future and thus lock the profit. Similarly, if equation 9 is not true, we may short the implied dividend from options as in equation 7 and long the implied dividend from the dividend swap/future.

Scenario		2008		2009	
Reb. Rate	Com. Fee	Eq. 8	Eq. 9	Eq. 8	Eq. 9
0 bps	0 bps	2.43%	0.14%	6.39%	0.11%
25 bps	0 bps	0.28%	0.14%	3.61%	0.11%
50 bps	0 bps	0.00%	0.14%	1.99%	0.11%
0 bps	25 bps	2.01%	0.14%	5.49%	0.08%
0 bps	50 bps	1.62%	0.14%	4.65%	0.08%
0 bps	100 bps	0.81%	0.08%	3.45%	0.08%
25 bps	25 bps	0.14%	0.14%	3.14%	0.08%

Table 4. Percentage of Arbitrage in Different Scenarios

In the arbitrage test, we use the market-observed bid-ask spread and fix the borrowing lending rate spread at 50 bps in all the scenarios. The short-selling rebate rate and commission fee are set to different levels in different scenarios. We notice that most arbitrage opportunities are wiped away by the bid-ask spread and the borrowing lending rate spread. Table

4 summarizes the percentage of observations that break the inequality in equation 8 or equation 9, or in other words, the arbitrage opportunities, in different scenarios for year 2008 and 2009.

As we expect, the arbitrage opportunities decrease when the short-selling rebate rate and the commission fee increase, but some arbitrage opportunities seem to persist even when the rebate rate and commission fee are already reasonably high. We scrutinize the time distribution of the persistent arbitrage opportunities in Table 5 and find that the arbitrages related to equation 9 scatter across months and last for only a very short period (within 1 hour). However, the arbitrages related to equation 8 cluster in October 2008 and last for days. Interestingly, the arbitrages seem to cluster when the SX5E index drops below 3000.

These arbitrage opportunities are actually driven by the high demand for put options in a bear market. When most investors expect the market to continue going down, the demand for put options rises, pushing up the put option market price to a higher level than its theoretical price. In this case, the put-call parity is temporarily broken, and a strategy that shorts the put option will get a higher implied index dividend. The higher implied index dividend does not reflect market expectation of higher dividend payment, but comes only from the temporary deviation of the put option from its theoretical value.

Month	2008		2009	
	Eq. 8	Eq. 9	Eq. 8	Eq. 9
Jan 2008	0.00%	0.00%	0.00%	0.00%
Feb 2008	0.00%	0.00%	0.00%	0.00%
Mar 2008	0.00%	0.00%	0.11%	0.00%
Apr 2008	0.00%	0.00%	0.00%	0.00%
May 2008	0.00%	0.00%	0.00%	0.00%
Jun 2008	0.00%	0.06%	0.00%	0.08%
Jul 2008	0.00%	0.00%	0.00%	0.00%
Aug 2008	0.00%	0.00%	0.00%	0.00%
Sep 2008	0.03%	0.08%	0.06%	0.03%
Oct 2008	2.40%	0.00%	6.22%	0.00%
Total	2.43%	0.14%	6.39%	0.11%

Table 5. Time Distribution of Arbitrage Opportunities

3.3 Implied Dividend Test for Index Future

The index future contract also reflects the market expectation of the index dividend, as equation 10 shows:

$$FV\left(\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})\right) = FV(S) - F \quad (10)$$

F : the index future price

Similar to the test we do for put-call parity, we calculate the implied dividend from the index future contract and compare it with the implied dividend calculated from the index dividend swap. We define the percentage form of absolute value of the difference and monetary form of absolute value of the difference, and report the quantiles in Table 6. Since the index future for year 2009 had not start trading when this version of paper was written, we have the data only for year 2008.

2008		
Quantile	percentage	monetary
10%	0.33%	0.53
30%	0.80%	1.27
50%	1.29%	2.05
75%	1.96%	3.11
90%	2.59%	4.10
95%	3.12%	4.13

Table 6. Quantiles of Absolute Value of Difference

From Table 6, we note that the implied dividend calculated from the index future contract is quite close to the implied dividend calculated from the index dividend swap. Hence, the future market is very efficient during our observation period.

3.4 No-Arbitrage Test for Index Future

The no-arbitrage test for index future is similar to the test for put-call parity. We consider the same set of market frictions and similar inequality as in equations 8 and 9. The result is reported in Table 7.

Scenario		2008	
Reb. Rate	Com. Fee	Eq. 8	Eq. 9
0 bps	0 bps	7.77%	0.75%
25 bps	0 bps	0.26%	0.75%
50 bps	0 bps	0.00%	0.75%
0 bps	25 bps	0.26%	0.26%
0 bps	50 bps	0.00%	0.09%
0 bps	100 bps	0.00%	0.04%
25 bps	25 bps	0.00%	0.26%

Table 7. Percentage of Arbitrage in Different Scenarios

As we see, very few arbitrage opportunities exist when market frictions are considered. Even if some arbitrages emerge, they are not persistent and disappear within 1 hour.

4 Index Dividend Swap Hedging Strategy

Since the index dividend is no more than a sum of its member companies' dividend, we introduce the simple strategy of hedging the individual member company's dividend shock first. As long as the individual member company's dividend shock can be fully hedged, the index dividend hedging will be no

more than a combination of hedging portfolios for all the member companies.

4.1 Hedge Individual Company's Dividend Shock

For most individual companies, options are actively traded. However, future contracts may not be available. Hence, to hedge the individual company's dividend swap/future, we can use the put-call parity. A long position in dividend swap/future has the payoff at maturity date T as follows:

$$Payoff_{DivSwap} = \sum_{j=1}^N Div(t_j) - K^{Div} \quad (11)$$

Here, the indicator j denotes the times of dividends the individual company pays during the contract period. Since companies may pay quarterly dividends or semiannual dividends, there may be multiple times of dividend payments during the contract period. A long position in dividend swap/future is actually a long position in actual dividend to be realized and a short position in the implied dividend. At the same time, a synthetic short position in actual dividend to be realized plus a synthetic long position in the implied dividend can be created by put-call parity, and the portfolio has payoff at maturity date T as follows:

$$Payoff_{PCP} = FV(P_t) + FV(S_t) - FV(C_t) - K^S - FV\left(\sum_{j=1}^N Div(t_j)\right) \quad (12)$$

The first four components on the right hand side construct the future value of implied dividend, and the last component on the right hand side is the future value of the actual dividend to be realized during the contract period. The uncertainty in equations 11 and 12 both come from the actual dividend to be realized (i.e. $\sum_{j=1}^N Div(t_j)$). As a result, we may use the uncertainty in equation 12 to offset the uncertainty in equation 11 and to hedge the dividend swap/future. The hedge ratio for company i is defined in equation

13:

$$\delta_i = \frac{\sum_{j=1}^N Div(t_{ij})}{\widehat{FV}(\sum_{j=1}^N Div(t_{ij}))} = \frac{1}{1 + \sum_{j=1}^N \widehat{w}_{ij} \widehat{\tau}_{ij} \widehat{r}_{ij}} \quad (13)$$

\widehat{w}_{ij} : the estimated weights for the j th dividend payment made by the i th company, so $\widehat{w}_{ij} = Div(t_{ij}) / (\sum_{l=1}^N Div(t_{il}))$

$\widehat{\tau}_{ij}$: the length of the estimated discount period for the j th dividend payment, $\widehat{\tau}_{ij} = T - t_{ij}$

\widehat{r}_{ij} : the estimated discount rate related to the $\widehat{\tau}_{ij}$

The estimation of parameters listed above is usually based on the previous year's dividend schedule and the current interest rate term structure model. They are good estimations, because for the same index, the individual member companies usually pay dividends based on their own fixed schedules each year. For example, if we compare the cumulative index dividend payment

schedule (in percentage form) for years 2005, 2006 and 2007 in Figure 3, it is obvious that they are quite close to each other, verifying that the individual member companies usually have a fixed schedule for dividend payments.

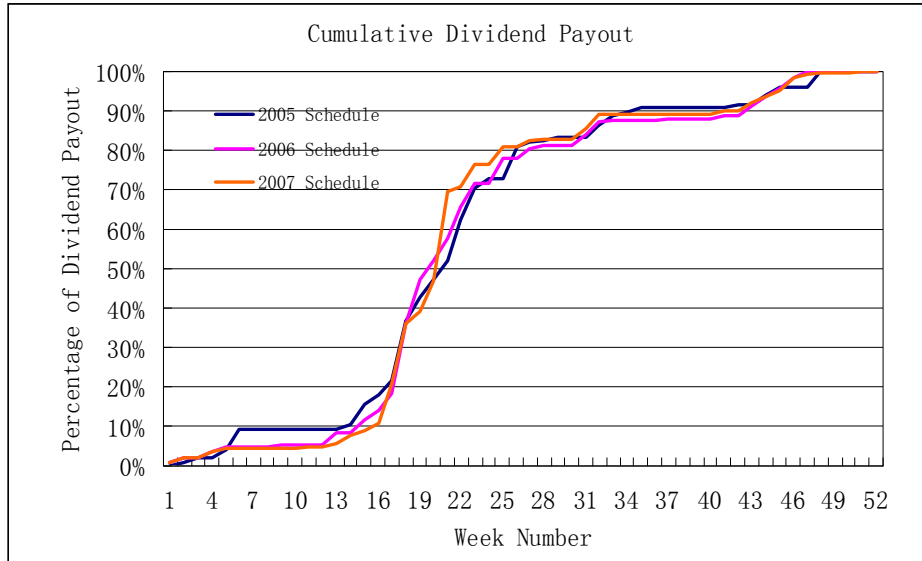


Fig 3. Cumulative Dividend Payment Schedule

As a result, we need to long δ_i units of put-call parity portfolio as in equation 12 to hedge one unit of company i 's dividend swap/future.

4.2 Hedge Index Dividend Shock

An index dividend swap/future is no more than a combination of all members' individual dividend swaps/futures. As a result, a direct and perfect hedging strategy can be created by hedging all the members' dividends separately

using the method we propose above. However, this hedging strategy is not practical because the hedging cost will be extremely high and it is very time-consuming to manage this hedging position. Since options on member companies are usually not as liquid as the options on the whole index, the bid-ask spread is higher. At the same time, it is time-consuming to calculate and manage the number of options investors need to buy or sell to create the hedging strategy for 50 companies. On the other hand, the hedging portfolio with options on an individual company is more volatile because such options usually incorporate more volatility, which is related to the company's idiosyncratic risk.

The practical hedging strategy we propose to hedge the index dividend swap/future is to use the put-call parity or index futures. According to our observation, index futures usually have lower bid-ask spread, but they are not available in the early stage of index dividend swap/future (e.g., the index future contract with a maturity date of 12/18/2008 starts trading on 3/25/2008, while the index dividend swap for 2008 starts on 12/27/2007). We introduce this hedging strategy using put-call parity below, and the hedging strategy with index futures can be deduced easily. Similar to the definition of hedge ratio δ_i for an individual company's dividend, the hedge ratio Ω for

index dividend is defined as follows:

$$\Omega = \frac{\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})}{\widehat{FV}(\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij}))} \quad (14)$$

Proposition 1 *The index dividend hedge ratio Ω is a close approximation of the weighted average of individual company dividend hedge ratio δ_i with*

$$\text{the weight } W_i = \frac{\sum_{j=1}^{N_i} Div(t_{ij})}{\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})}, \text{ i.e. } \Omega \approx \sum_{i=1}^M W_i \delta_i$$

Proof. According to the definition of Ω ,

$$\Omega = \frac{\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})}{\widehat{FV}(\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij}))} = \frac{\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})}{\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})(1+\widehat{r}_{ij}\widehat{\tau}_{ij})}$$

Divide $\sum_{i=1}^M \sum_{j=1}^{N_i} Div(t_{ij})$ on numerator and denominator and we get

$$\begin{aligned} \Omega &= 1/\{1 + \sum_{i=1}^M \sum_{j=1}^{N_i} [\frac{Div(t_{ij})}{\sum_{k=1}^M \sum_{l=1}^{N_k} Div(t_{kl})} \widehat{r}_{ij}\widehat{\tau}_{ij}]\} \\ &= 1/\{1 + \sum_{i=1}^M \sum_{j=1}^{N_i} [\frac{\sum_{l=1}^{N_i} Div(t_{il})}{\sum_{k=1}^M \sum_{l=1}^{N_k} Div(t_{kl})} \frac{Div(t_{ij})}{\sum_{l=1}^{N_i} Div(t_{il})} \widehat{r}_{ij}\widehat{\tau}_{ij}]\}. \end{aligned}$$

As before, we define that

$$\widehat{w}_{ij} = \frac{Div(t_{ij})}{\sum_{l=1}^{N_i} Div(t_{il})} \text{ and } W_i = \frac{\sum_{l=1}^{N_i} Div(t_{il})}{\sum_{k=1}^M \sum_{l=1}^{N_k} Div(t_{kl})}.$$

Substitute \widehat{w}_{ij} and W_i into the Ω formula:

$$\begin{aligned} \Omega &= 1/\{1 + \sum_{i=1}^M W_i [\sum_{j=1}^{N_i} \widehat{w}_{ij}\widehat{r}_{ij}\widehat{\tau}_{ij}]\} \\ &\approx 1 - \sum_{i=1}^M W_i [\sum_{j=1}^{N_i} \widehat{w}_{ij}\widehat{r}_{ij}\widehat{\tau}_{ij}] \\ &= \sum_{i=1}^M W_i (1 - \sum_{j=1}^{N_i} \widehat{w}_{ij}\widehat{r}_{ij}\widehat{\tau}_{ij}) \end{aligned}$$

$$\begin{aligned}
&\approx \sum_{i=1}^M W_i \frac{1}{1 + \sum_{j=1}^{N_i} \widehat{w}_{ij} \widehat{r}_{ij} \widehat{\tau}_{ij}} \\
&= \sum_{i=1}^M W_i \delta_i \quad \blacksquare
\end{aligned}$$

Proposition 2 *Given that the dividend schedule is consistent with the schedule estimation and no shock in interest rate occurs, the hedging strategy created by hedge ratio Ω is a perfect hedging when all the member companies have proportional shock to their dividend payment, that is, $\frac{\Delta Div(t_{ij})}{Div(t_{ij})} = cons. \forall i, j$.*

Proof. Let the fully hedged portfolio be Π . When dividend shock occurs, the change in the payoff of hedged portfolio is as follows:

$$\begin{aligned}
\Delta Payoff_{\Pi} &= \Omega \cdot \Delta Payoff_{PCP} + \Delta Payoff_{DivSwap} \\
&= -\Omega \cdot FV\left(\sum_{i=1}^M \sum_{j=1}^{N_i} \Delta Div(t_{ij})\right) + \sum_{i=1}^M \sum_{j=1}^{N_i} \Delta Div(t_{ij}) \\
&= \left\{ FV\left(\sum_{i=1}^M \sum_{j=1}^{N_i} \Delta Div(t_{ij})\right) \right\} \left\{ \frac{\sum_{i=1}^M \sum_{j=1}^{N_i} \Delta Div(t_{ij})}{FV\left(\sum_{i=1}^M \sum_{j=1}^{N_i} \Delta Div(t_{ij})\right)} - \Omega \right\} \\
&= \left\{ FV\left(\sum_{i=1}^M \sum_{j=1}^{N_i} \Delta Div(t_{ij})\right) \right\} \left\{ \frac{1}{1 + \sum_{i=1}^M \sum_{j=1}^{N_i} \frac{\Delta Div(t_{ij})}{\sum_{k=1}^M \sum_{l=1}^{N_k} \Delta Div(t_{kl})} r_{ij} \tau_{ij}} - \frac{1}{1 + \sum_{i=1}^M \sum_{j=1}^{N_i} \frac{Div(t_{ij})}{\sum_{k=1}^M \sum_{l=1}^{N_k} Div(t_{kl})} \widehat{r}_{ij} \widehat{\tau}_{ij}} \right\}
\end{aligned}$$

If the actual dividend schedule is consistent with the schedule estimation used in the Ω calculation and no shock in interest rate occurs, the following equalities hold: $r_{ij} = \widehat{r}_{ij}$ and $\tau_{ij} = \widehat{\tau}_{ij}$. With the assumption of proportional dividend shock, we have

$$\frac{\Delta Div(t_{ij})}{\sum_{k=1}^M \sum_{l=1}^{N_k} \Delta Div(t_{kl})} = \frac{Div(t_{ij})}{\sum_{k=1}^M \sum_{l=1}^{N_k} Div(t_{kl})}$$

Substitute in all the equalities, and we get

$\Delta Payof f_{\Pi} = 0$, and thus it is a perfect hedging. ■

Proposition 3 *Given that the dividend schedule is consistent with the schedule estimation and no shock in interest rate occurs, the effectiveness of the hedging strategy created by hedge ratio Ω is decided by the dispersion of individual company dividend hedge ratio δ_i when non-proportional shock occurs to the individual company's dividend payment.*

Proof. A non-proportional dividend shock can be seen as a combination of shocks in the single dividend payment. As a result, we can examine only the hedging strategy for the shock occurring to the single dividend payment that company i makes, that is,

$$\sum_{j=1}^N \Delta Div(t_{ij}) \neq 0 \text{ and } \Delta Div(t_{kl}) = 0, \forall k \neq i$$

In this case, the fully hedged portfolio's payoff has a change as follows:

$$\begin{aligned} \Delta Payof f_{\Pi} &= \Omega \cdot \Delta Payof f_{PCP} + \Delta Payof f_{DivSwap} \\ &= -\Omega \cdot FV\left(\sum_{j=1}^N \Delta Div(t_{ij})\right) + \sum_{j=1}^N \Delta Div(t_{ij}) \\ &= FV\left(\sum_{j=1}^N \Delta Div(t_{ij})\right) \left\{ \frac{\sum_{j=1}^N \Delta Div(t_{ij})}{FV\left(\sum_{j=1}^N \Delta Div(t_{ij})\right)} - \Omega \right\} \\ &\approx FV\left(\sum_{j=1}^N \Delta Div(t_{ij})\right) \left\{ \delta_i - \sum_{i=1}^M W_i \delta_i \right\} \end{aligned}$$

As a result, the fully hedged portfolio will be affected when non-proportional shock occurs, and the effect is proportional to the difference between δ_i and Ω . Since Ω is a close approximation of the weighted average of δ_i , the dif-

ference between δ_i and Ω is captured by the dispersion of the δ_i set. Large dispersion will, on average, lead to a greater difference between δ_i and Ω and thus cause larger change on the hedged portfolio's payoff. ■

4.3 Index Dividend Swap Hedging Performance

To evaluate the performance of the hedging strategy we propose in this section, we create two separate virtual hedging portfolios for the 2008 and 2009 index dividend swap/future. The monetary profits of the fully hedged portfolio (i.e., the whole portfolio containing a long position in index dividend swap/future and a hedging position created by put-call parity) and the unhedged portfolio are compared in Figure 4 and 5.

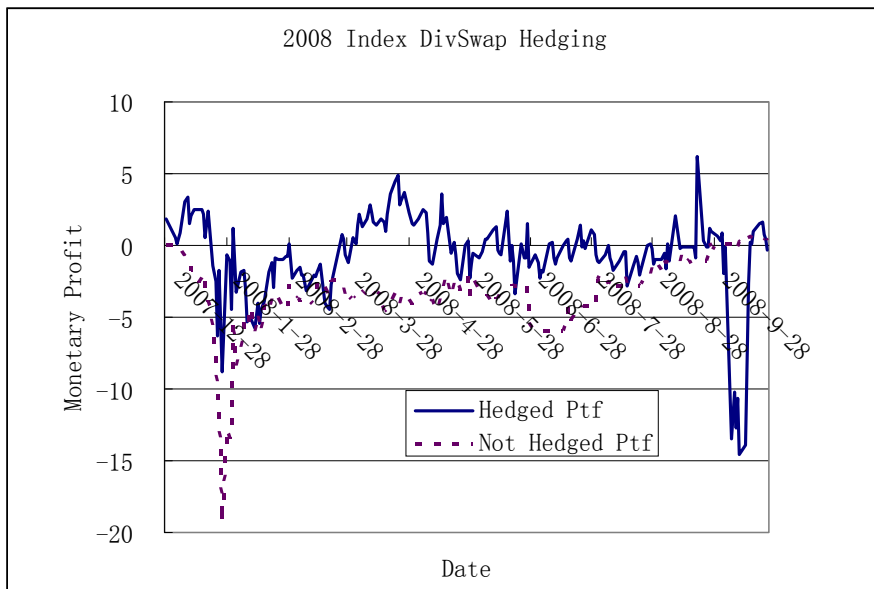


Fig 4. Hedging Performance of the 2008 Index Dividend Swap

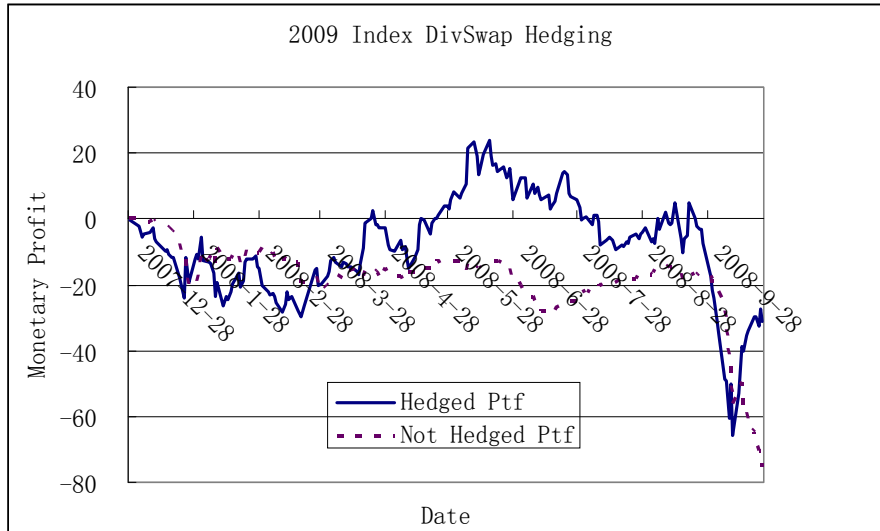


Fig 5. Hedging Performance of the 2009 Index Dividend Swap

Note that the market frictions are not included in the profit calculation. The profits calculated in Figure 4 and Figure 5 are mark-to-market profits, that is, the profits the investors can obtain by liquidating the whole portfolio (index dividend swap/future and the hedging position) immediately at market price. Hence, the profits are volatile and sensitive to the market efficiency. For example, in section 3 our empirical test shows that the index option market is not efficient in the short term during October 2008. And, congruently, we notice that the hedged portfolio profits for both the 2008 and 2009 index dividend swap drop significantly during that period. This is because the put options are overpriced and the hedging portfolio has a short position in the put option. This overpricing reverses in the middle of October 2008 for put options whose maturity date are on the end of 2008, so the 2008 hedged

portfolio profit reverses to zero. The put options whose maturity dates are at the end of 2009 tend to reverse to their fair value but haven't completed, so the hedged portfolio profit is still negative. But even in this case, the hedged portfolio greatly outperforms the unhedged portfolio because there is a large shock to the 2009 index dividend expectation, that is, from about 160 at the beginning of 2008 to about 90 recently. The market overall expects a large dividend cut in year 2009 and thus the hedging helps. Last but not least, if the investors hold the hedging position until maturity, the put-call parity must hold on the maturity date, so the hedged portfolio profit must be zero then no matter how volatile it was before the maturity. In other words, the hedged portfolio has mark-to-market risk during the contract period, but it should be risk-free if it is held to maturity.

5 Conclusion and Discussion

In this paper, we are motivated to investigate index dividend trading. This is a quite new topic in that the index dividend is traded independently from the index itself. Recent financial crisis provides us with a unique opportunity to dig deeper into this field. The large shock to 2009 index dividends calls for more research into dividend investment hedging. We start our research by testing the efficiency of dividend-relevant financial products such as op-

tions and futures. Our tests are conducted in two steps: first, we calculate the implied dividend by put-call parity and index futures and check whether they are consistent with the implied dividend calculated from index dividend swap/future (an instrument that directly trades the index dividend) without considering market friction. We find that the result is mixed. The market data show that the implied dividends calculated from different instruments are consistent statistically for year 2008, but the results are not as good for year 2009. In the second step, we test the non-arbitrage relationship with the existence of market friction. Our finding is similar to much previous research on the put-call parity test, namely, that very few arbitrage opportunities appear when market friction are considered. However, we do discover some persistent arbitrage opportunities when the market tumbles and the imbalance between supply and demand drives the option price to deviate from the fair value. The mispricing tends to reverse in the short term and then most arbitrages are eliminated. In our test, we are convinced that the bid-ask spread and the borrowing-lending rate spread are the most important friction for put-call parity and index future pricing model, and they contribute to wiping away over 90% of arbitrages.

Based on our empirical result, we propose two hedging strategies. The first is a perfect hedging strategy, but it incurs very high cost and bears high volatility. The second is more practical, but it is not a perfect hedging

strategy in general. We prove that it is a perfect hedging only when all the individual member companies in the index have proportional dividend shock. When non-proportional shock occurs, we also obtain an estimation of the hedging error that depends on the dispersion of all dividend discount factors. We create virtual hedging portfolios for 2008 and 2009 index dividend swap/future separately and compare the fully hedged portfolio profits with the unhedged portfolio profits. The hedging strategy we propose is proved to be very effective and the fully hedged portfolio greatly outperforms the unhedged when large dividend shock occurs. We also find that the hedging strategies created by put-call parity or index futures have mark-to-market risk before the contract maturity, so the market price of the fully hedged portfolio is not always close to zero, especially when the market is suddenly disturbed and becomes inefficient in the short term. But luckily, the fully hedged portfolio profit will always converge to zero on the maturity date, thus verifying the hedging and holding to maturity strategy.

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